USN

Fifth Semester B.E. Degree Examination, December 2012 Information Theory and Coding

Time: 3 hrs. Max. Marks:100

Note: Answer FIVE full questions, selecting at least TWO questions from each part.

PART – A

- a. Define the following with respect to information theory: i) Self information ii) Entropy iii) Rate of information iv) Mutual information. (04 Marks)
 - b. Prove that the entropy of the following probability distribution function is $2 \left(\frac{1}{2}\right)^{n-2}$.

(08 Marks)

Symbols:	\mathbf{x}_1	X 2	X3	 X _{n-1}	Xn
Probability of $(x = x_i)$:	1	1	1	 1	1
	$\frac{1}{2}$	$\frac{-}{4}$	8	$\overline{2^{n-1}}$	$\frac{1}{2^{n-1}}$

- c. A sample space of events is shown in the diagram below with probability $P = \left\{ \frac{1}{5}, \frac{4}{15}, \frac{8}{15} \right\}$,
 - i) Evaluate average uncertainity associated with the scheme.
 - ii) Average uncirtainity pertaining to the following probability scheme:

$$P[A/M = B \cup C], P[B/M, C/M]$$

iii) Verify additive rule.

(08 Marks)

2 a. Given the model of a Markoff source in Fig. Q2 (a)

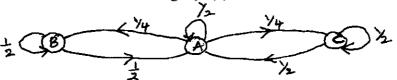


Fig. Q2 (a)

- Find i) State probability ii) Entropy of first order and second order source
 - iii) Efficiency and redundancy of first order source
- iv) Find rate of information if $r_s = 1$ sym/sec. (10 Marks)
- b. Design an encoder using Shannons encoding algorithm for a source having six symbols and probability statistics $P = \left\{ \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{32} \right\}$ (10 Marks)
- 3 a. Consider a source with 8 alphabets A to H with respective probabilities of 0.22, 0.20, 0.18, 0.15, 0.10, 0.08, 0.05, 0.02
 - i) Construct a binary compact code and determine coding efficiency using Huffman coding algorithm.
 - ii) Construct ternary Huffman code and determine efficiency of the code. (10 Marks)
 - b. Prove that H(X/Y) = p.H(X) for a binary erasure channel.

(05 Marks)

c. Given the following channel matrix find the channel capacity:

 $P(Y/X) = \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.2 & 0.8 \end{bmatrix}$ (05 Marks)

- 4 a. Define i) Differential entropy
- ii) Shannon's limit

- (02 Marks)
- b. Prove that for an infinite bandwidth signal energy to noise ratio $\frac{E}{\eta}$ approaches a limiting value.
- c. A black and white TV picture may be viewed as consisting of 3×10^5 elements, each of which occupies 10 distinct brightness levels with equal probability. Assume rate of transmission as 30 picture frames per sec and SNR = 30 dB. Using channel capacity theorem compute minimum bandwidth to error free transmission of video signal. (06 Marks)
- d. Prove that $\lim_{B\to\infty} C = 1.44 \frac{S}{\eta}$.

(06 Marks)

PART - B

5 a. Consider a systematic (7, 4) linear block code, the parity check matrix,

$$\mathbf{P} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

- i) Find all possible code words.
- ii) Draw encoding circuit.
- iii) A single bit error has occurred in each of the following code words given:

$$R_A = [0 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0], \qquad R_B = [1 \ 0 \ 1 \ 1 \ 1 \ 0 \ 0]$$

Detect and correct these errors

iv) Draw syndrome computation circuit.

(12 Marks)

- b. Find generator matrix G and H-matrix for a linear block code with $d_{min} = 3$ and message block size of 8 bits. (04 Marks)
- c. Test hamming bound of (7, 4) hamming code and show that it is a perfect code. (04 Marks)
- 6 a. Design an encoder for (7, 4) binary cyclic code generated by $G(x) = 1 + x + x^3$ and verify its operation using message vectors $(1\ 0\ 0\ 1)$ and $(1\ 0\ 1\ 1)$. Also verify the code obtained using polynomial arithmetic. (10 Marks)
 - b. For a (7, 4) cyclic code with received vector Z is 1 1 1 0 1 0 1, with the generator polynomial $G(x) = 1 + x + x^3$. Draw the syndrome computation circuit and correct, the error in the received vector.
- Write short notes on: a. Shortened cyclic codes
- b. Golav codes.
- c. BCH codes.
- d. RS codes.

(20 Marks)

- 8 a. For the convolution encoder shown in Fig. Q8 (a).
 - i) Find impulse response and hence calculate the output produced by the information sequence (1 0 1 1 1).
 - ii) Write the generator polynomials of the encoder and recompute the output of the input of (i) and compare with that of (ii). (08 Marks)

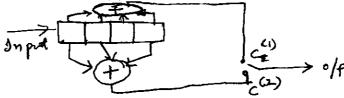


Fig. Q8 (a)

b. Consider a (3, 1, 2) convolution encoder with $g^{(1)} = 1 \cdot 1 \cdot 0$ and $g^{(2)} = 1 \cdot 0 \cdot 1$, $g^{(3)} = 1 \cdot 1 \cdot 1$. Draw encoder block diagram, find generator matrix. Find code vector corresponding to information sequence $D = 1 \cdot 1 \cdot 1 \cdot 0 \cdot 0$ using time and frequency domain approach. Draw state diagram and code tree.

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